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UNIVERSITY COLLEGE TATI

UNIVERSITY COLLEGE TATI (UC TATI)

FINAL EXAMINATION QUESTION BOOKLET

COURSE CODE	:	BMT 3043
COURSE	:	SIGNAL THEORY AND APPLICATION
SEMESTER/SESSION	:	2-2021/2022
DURATION	:	3 HOURS

Instructions:

1. This booklet contains **4** questions. Answer **ALL** questions.
2. All answers should be written in answer booklet.
3. Write legibly and draw sketches wherever required.
4. If in doubt, raise up your hands and ask the invigilator.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

THIS BOOKLET CONTAINS 16 PRINTED PAGES INCLUDING COVER PAGE

QUESTION 1

a) The square waveform of Figure 1 is an odd function waveform. This waveform also half-wave symmetry. Compute all coefficients to verify this waveform. assume that $\omega = 1$ (9 Marks)

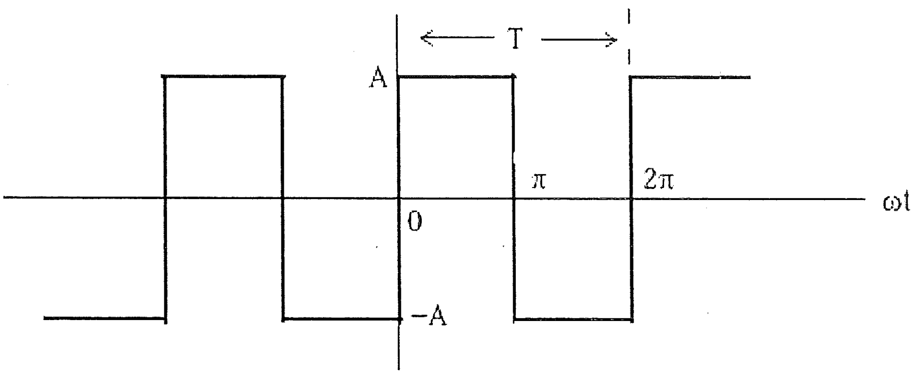

Figure 1

b) Compute the first five (5) components of the trigonometric Fourier series for the waveform shown in Figure 2. Assume $\omega = 1$ (8 Marks)

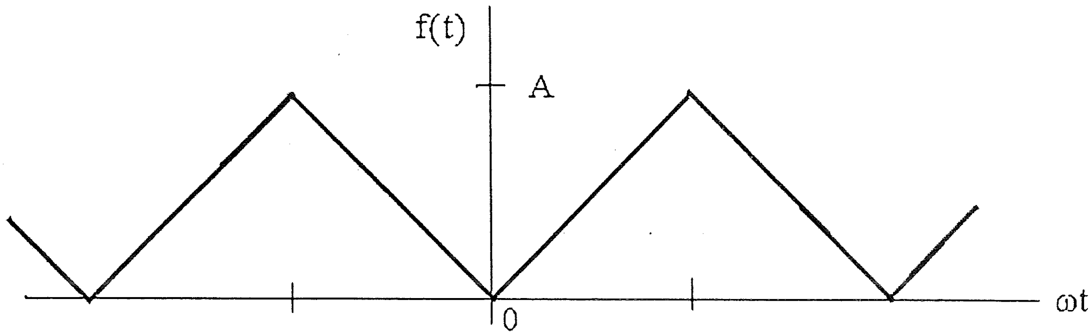

Figure 2

QUESTION 2

a) Derive the Fourier transform of $f(t) = e^{-a|t|}$ using

i. Fourier transform definition (3 marks)

ii. By substitution into the Laplace transform equivalent (3 marks)

b) The symmetric rectangular pulse waveform $f(t) = [u_o(t+T) - u_o(t-T)]$ shows in Figure 3. Compute the Fourier transform for this rectangular pulse transform.
(5 marks)

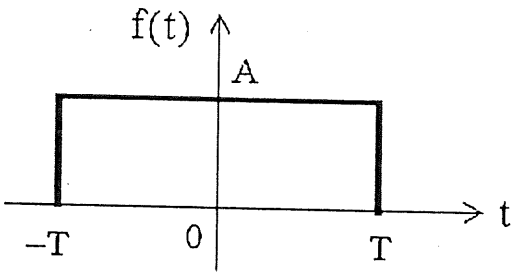

Figure 3

c) Derive the Fourier transform of

$f(t) = A[u_o(t+3T) - u_o(t+T) + u_o(t-T) - u_o(t-3T)]$ (4 Marks)

d) Figure 4 shows a simple RL circuit. Using the Fourier transform method, and the system function $H(\omega)$, compute $v_L(t)$. Assume $i_L(0^-)$.
(10 Marks)

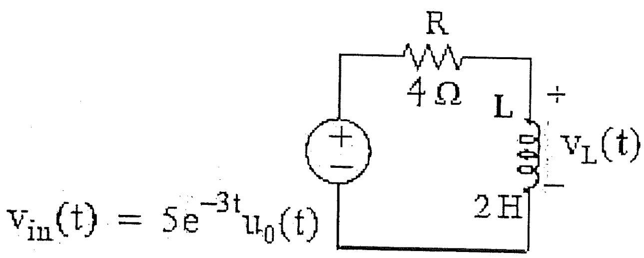

Figure 4

e) Analog filter is defined over a continuous range of frequency. They are classified into four (4) types: low-pass filter, high-pass filter, band-pass filter, and band-elimination (stop-band).

i. Explain all four (4) types of analog filters. (4 Marks)

ii. Draw all four (4) types of analog filters with magnitude characteristics (6 Marks)

QUESTION 3

a) Compute the Laplace transform of the following time domain functions

i. $(2t^2 - 5t \quad + 4)u_o(t-3)$ (2 Marks)

ii. $(3\sin 5t)u_o(t)$ (2 Marks)

iii. $4e^{-3t}(cos2t)u_o\,(t)$ (2 Marks)

b) The equation below is one of the Laplace transform expressions using the partial fraction expansion method

$$F_2(s) = \frac{3s^2 + 2s + 5}{s^3 + 12s^2 + 44s + 48}$$

i. Derive the equation above using the partial fraction expression (6 marks)

ii. Formulate the time domain function f_2 (t) corresponding to $F_2(s)$ (2 marks)

c) Figure 5 shows RC circuit and can be transformed from time to the complex frequency circuit. Use the Laplace transform method and Kirchoff's Voltage Law (KCL) to produce the voltage $V_C(t)$ across the RC circuit's capacitor. Given that $V_C(0^-) = 6$ V.

(6 marks)

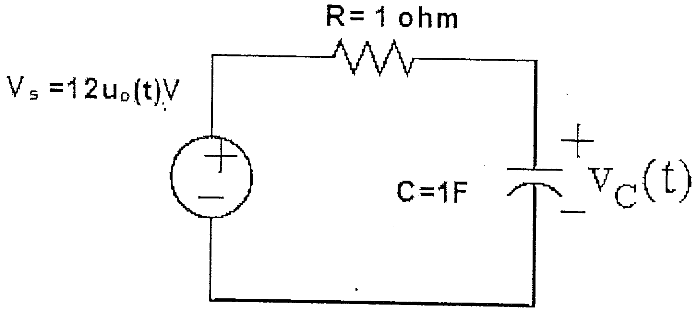

Figure 5

d) Figure 6 shows serial parallel circuit for complex impedance Z(s) and complex admittance Y(s). **All values are in Ω (ohms).**

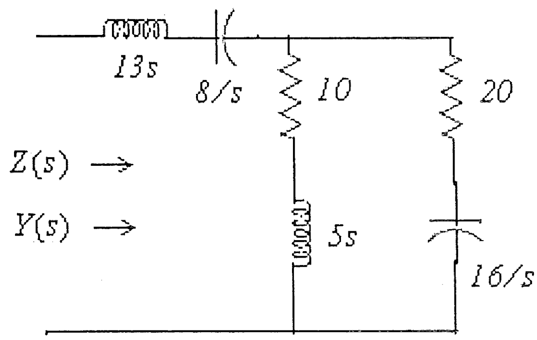

Figure 6

i. Identify complex impendence Z(s) (4 Marks)

ii. Formulate complex admittance Y(s). (1 Mark)

QUESTION 4

a) Derive the z-transform of the signal below (2 Marks)

$$x[n] = \frac{1}{2}^{n} u[n]$$

b) Using the partial fraction expansion method, compute the Inverse Z transform of the equation below. (6 Marks)

$$F(z) = \frac{12z}{(z+1)(z-1)^2}$$

c) A discrete time system is described by the difference equation below

$$y[n] + y[n-1] = x[n]$$

where

$$y[n] = 0 \text{ for } n<0$$

From equation above:

i. Identify transfer function. H(z). (2 Marks)

ii. Compute the discrete time impulse response h[n]. (2 Marks)

iii. Identify the response when the input is $x[n] = 10$ for $n \geq 0$ (5 Marks)

d)

Given the transfer function for G(s) is,

$$G_S = \frac{3s^2 + 5s + 7}{s^2 + 4s + 6}$$

Derive the magnitude-square function, $A^2(\omega)$ (6 Marks)

--End of question--

Attachment

List of integrals of exponential functions

Integral Rule	General Rule
$\int \cos x\, dx = \sin x + C$	$\int \cos(ax+b)dx = \frac{1}{a}\sin(ax+b) + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sin(ax+b)dx = -\frac{1}{a}\cos(ax+b) + C$
$\int \tan x\, dx = -\ln\left\lvert \cos x\right\rvert + C$	$\int \tan(ax+b)dx = -\frac{1}{a}\ln\left\lvert \cos(ax+b)\right\rvert + C$
$\int \text{cotan}\, x\, dx = \ln\left\lvert \sin x\right\rvert + C$	$\int \text{cotan}(ax+b)dx = \frac{1}{a}\ln\left\lvert \sin(ax+b)\right\rvert + C$
$\int \sec x\, dx = \ln\left\lvert \sec x + \tan x\right\rvert + C$	$\int \sec(ax+b)dx = \frac{1}{a}\ln\left\lvert \sec(ax+b) + \tan(ax+b)\right\rvert + C$
$\int \text{cosec}\, x\, dx = -\ln\left\lvert \text{cosec}\, x + \text{cotan}\, x\right\rvert + C$	$\int \text{cosec}(ax+b)dx = -\frac{1}{a}\ln\left\lvert \text{cosec}(ax+b) + \text{cotan}(ax+b)\right\rvert + C$

$$\int e^{x}\,\mathrm{d}x = e^{x}$$

$$\int e^{cx}\,\mathrm{d}x = \frac{1}{c}e^{cx}$$

$$\int a^{cx}\,\mathrm{d}x = \frac{1}{c\cdot\ln a}a^{cx} \text{ for } a>0,\ a\neq 1$$

$$\int xe^{cx}\,\mathrm{d}x = \frac{e^{cx}}{c^2}(cx-1)$$

$$\int x^2e^{cx}\,\mathrm{d}x = e^{cx}\left(\frac{x^2}{c}-\frac{2x}{c^2}+\frac{2}{c^3}\right)$$

$$\int x^ne^{cx}\,\mathrm{d}x = \frac{1}{c}x^ne^{cx}-\frac{n}{c}\int x^{n-1}e^{cx}\mathrm{d}x = \left(\frac{\partial}{\partial c}\right)^n\frac{e^{cx}}{c}$$

$$\int \frac{e^{cx}}{x}\,\mathrm{d}x = \ln|x| + \sum_{n=1}^{\infty}\frac{(cx)^n}{n\cdot n!}$$

$$\int \frac{e^{cx}}{x^n}\,\mathrm{d}x = \frac{1}{n-1}\left(-\frac{e^{cx}}{x^{n-1}}+c\int\frac{e^{cx}}{x^{n-1}}\,\mathrm{d}x\right) \qquad (\text{for } n\neq 1)$$

$$\int e^{cx}\ln x\,\mathrm{d}x = \frac{1}{c}e^{cx}\ln|x| - \operatorname{Ei}(cx)$$

$$\int e^{cx}\sin bx\,\mathrm{d}x = \frac{e^{cx}}{c^2+b^2}(c\sin bx - b\cos bx)$$

$$\int e^{cx}\cos bx\,\mathrm{d}x = \frac{e^{cx}}{c^2+b^2}(c\cos bx + b\sin bx)$$

$$\int e^{cx}\sin^n x\,\mathrm{d}x = \frac{e^{cx}\sin^{n-1}x}{c^2+n^2}(c\sin x - n\cos x) + \frac{n(n-1)}{c^2+n^2}\int e^{cx}\sin^{n-2}x\,\mathrm{d}x$$

$$\int e^{cx}\cos^n x\,\mathrm{d}x = \frac{e^{cx}\cos^{n-1}x}{c^2+n^2}(c\cos x + n\sin x) + \frac{n(n-1)}{c^2+n^2}\int e^{cx}\cos^{n-2}x\,\mathrm{d}x$$

$$\int xe^{cx^2}\,\mathrm{d}x = \frac{1}{2c}\,e^{cx^2}$$

$$\int e^{x^2}\,dx = e^{x^2}\left(\sum_{j=0}^{n-1} c_{2j}\,\frac{1}{x^{2j+1}}\right) + (2n-1)c_{2n-2}\int \frac{e^{x^2}}{x^{2n}}\,dx \quad \text{valid for } n>0,$$

where $c_{2j} = \dfrac{1\cdot 3\cdot 5\cdots(2j-1)}{2^{j+1}} = \dfrac{(2j)!}{j!\,2^{2j+1}}$.

$$\int \underbrace{x^{x^{\cdot^{\cdot^{x}}}}}_{m}\,dx = \sum_{n=0}^{m} \frac{(-1)^n(n+1)^{n-1}}{n!}\Gamma(n+1,-\ln x) + \sum_{n=m+1}^{\infty}(-1)^n a_{mn}\Gamma(n+1,-\ln x) \qquad (\text{for } x>0)$$

where $a_{mn} = \begin{cases} 1 & \text{if } n=0, \\ \frac{1}{n!} & \text{if } m=1, \\ \frac{1}{n}\sum_{j=1}^{n} j a_{m,n-j} a_{m-1,j-1} & \text{otherwise} \end{cases}$

and $\Gamma(x,y)$ is the Gamma Function

$$\int \frac{1}{ae^{\lambda x}+b}\,dx = \frac{x}{b} - \frac{1}{b\lambda}\ln\left(ae^{\lambda x}+b\right) \text{ when } b\neq 0,\ \lambda\neq 0, \text{ and } ae^{\lambda x}+b>0.$$

$$\int \frac{e^{2\lambda x}}{ae^{\lambda x}+b}\,dx = \frac{1}{a^2\lambda}\left[ae^{\lambda x}+b-b\ln\left(ae^{\lambda x}+b\right)\right] \text{ when } a\neq 0,\ \lambda\neq 0, \text{ and } ae^{\lambda x}+b>0.$$

Definite integrals

$$\int_0^1 e^{x\cdot\ln a+(1-x)\cdot\ln b}\,dx = \int_0^1 \left(\frac{a}{b}\right)^x\cdot b\,dx = \int_0^1 a^x\cdot b^{1-x}\,dx = \frac{a-b}{\ln a-\ln b} \text{ for}$$

$a>0,\ b>0,\ a\neq b$, which is the logarithmic mean

$$\int_0^\infty e^{ax}\,dx = \frac{1}{a}(a<0)$$

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}} \quad (a>0) \text{ (the Gaussian integral)}$$

$$\int_{-\infty}^\infty e^{-ax^2}\,dx = \sqrt{\frac{\pi}{a}} \quad (a>0)$$

$$\int_{-\infty}^\infty e^{-ax^2}e^{-2bx}\,dx = \sqrt{\frac{\pi}{a}}e^{\frac{b^2}{a}} \quad (a>0) \text{ (see Integral of a Gaussian function)}$$

$$\int_{-\infty}^\infty xe^{-a(x-b)^2}\,dx = b\sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^\infty x^2e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a^3}} \quad (a>0)$$

$$\int_0^\infty x^n e^{-ax^2}\,dx = \begin{cases} \frac{1}{2}\Gamma\left(\frac{n+1}{2}\right)/a^{\frac{n+1}{2}} & (n>-1, a>0) \\ \frac{(2k-1)!!}{2^{k+1}a^k}\sqrt{\frac{\pi}{a}} & (n=2k, k \text{ integer}, a>0) \\ \frac{k!}{2a^{k+1}} & (n=2k+1, k \text{ integer}, a>0) \end{cases} \quad \text{(!! is the double factorial)}$$

$$\int_0^\infty x^n e^{-ax}\,dx = \begin{cases} \frac{\Gamma(n+1)}{a^{n+1}} & (n>-1, a>0) \\ \frac{n!}{a^{n+1}} & (n=0,1,2,\ldots, a>0) \end{cases}$$

$$\int_0^\infty e^{-ax}\sin bx\,dx = \frac{b}{a^2+b^2} \quad (a>0)$$

Laplace Transform Properties and Theorems

	Property/Theorem	*Time Domain*	*Complex Frequency Domain*
1	Linearity	$c_1 f_1(t) + c_2 f_2(t) + \ldots + c_n f_n(t)$	$c_1 F_1(s) + c_2 F_2(s) + \ldots + c_n F_n(s)$
2	Time Shifting	$f(t-a)u_0(t-a)$	$e^{-as}F(s)$
3	Frequency Shifting	$e^{-as}f(t)$	$F(s+a)$
4	Time Scaling	$f(at)$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
5	Time Differentiation See also (2.18) through (2.20)	$\frac{d}{dt}f(t)$	$sF(s)-f(0^-)$
6	Frequency Differentiation See also (2.22)	$tf(t)$	$-\frac{d}{ds}F(s)$
7	Time Integration	$\int_{-\infty}^{t} f(\tau)d\tau$	$\frac{F(s)}{s}+\frac{f(0^-)}{s}$
8	Frequency Integration	$\frac{f(t)}{t}$	$\int_{s}^{\infty} F(s)ds$
9	Time Periodicity	$f(t+nT)$	$\frac{\int_{0}^{T} f(t)e^{-st}dt}{1-e^{-sT}}$
10	Initial Value Theorem	$\lim_{t\to 0} f(t)$	$\lim_{s\to\infty} sF(s) = f(0^-)$
11	Final Value Theorem	$\lim_{t\to\infty} f(t)$	$\lim_{s\to 0} sF(s) = f(\infty)$
12	Time Convolution	$f_1(t)*f_2(t)$	$F_1(s)F_2(s)$
13	Frequency Convolution	$f_1(t)f_2(t)$	$\frac{1}{2\pi j}F_1(s)*F_2(s)$

Laplace Transform Pairs for Common Functions

	f(t)	F(s)
1	$u_0(t)$	$1/s$
2	$tu_0(t)$	$1/s^2$
3	$t^n u_0(t)$	$\frac{n!}{s^{n+1}}$
4	$\delta(t)$	1
5	$\delta(t-a)$	e^{-as}
6	$e^{-at}u_0(t)$	$\frac{1}{s+a}$
7	$t^n e^{-at} u_0(t)$	$\frac{n!}{(s+a)^{n+1}}$
8	$\sin\omega t\ u_0(t)$	$\frac{\omega}{s^2+\omega^2}$
9	$\cos\omega t\ u_0(t)$	$\frac{s}{s^2+\omega^2}$
10	$e^{-at}\sin\omega t\ u_0(t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
11	$e^{-at}\cos\omega t\ u_0(t)$	$\frac{s+a}{(s+a)^2+\omega^2}$

Common Fourier transform pairs

$f(t)$	$F(\omega)$
$\delta(t)$	1
$\delta(t-t_0)$	$e^{-j\omega t_0}$
1	$2\pi\delta(\omega)$
$e^{-j\omega t_0}$	$2\pi\delta(\omega-\omega_0)$
$\mathrm{sgn}(t)$	$2/(j\omega)$
$u_0(t)$	$\frac{1}{j\omega}+\pi\delta(\omega)$
$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$
$\sin\omega_0 t$	$j\pi[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$
$e^{-at}u_0(t)$ $a>0$	$\frac{1}{j\omega+a}$ $a>0$
$te^{-at}u_0(t)$ $a>0$	$\frac{1}{(j\omega+a)^2}$ $a>0$
$e^{-at}\cos\omega_0 t u_0(t)$ $a>0$	$\frac{j\omega+a}{(j\omega+a)^2+\omega^2}$ $a>0$
$e^{-at}\sin\omega_0 t u_0(t)$ $a>0$	$\frac{\omega}{(j\omega+a)^2+\omega^2}$ $a>0$
$A[u_0(t+T)-u_0(t-T)]$	$2AT\frac{\sin\omega T}{\omega T}$

Properties and Theorems of the Z transform

Property / Theorem	Time Domain	Z transform
Linearity	$af_1[n] + bf_2[n] + \ldots$	$aF_1(z) + bF_2(z) + \ldots$
Shift of $x[n]u_0[n]$	$f[n-m]u_0[n-m]$	$z^{-m}F(z)$
Right Shift	$f[n-m]$	$z^{-m}F(z) + \sum_{n=0}^{m-1} f[n-m]z^{-n}$
Left Shift	$f[n+m]$	$z^{m}F(z) + \sum_{n=-m}^{-1} f[n+m]z^{-n}$
Multiplication by a^n	$a^n f[n]$	$F\left(\frac{z}{a}\right)$
Multiplication by e^{-naT}	$e^{-naT}f[n]$	$F(e^{aT}z)$
Multiplication by n	$nf[n]$	$-z\frac{d}{dz}F(z)$
Multiplication by n^2	$n^2 f[n]$	$z\frac{d}{dz}F(z) + z^2\frac{d^2}{dz^2}F(z)$
Summation in Time	$\sum_{m=0}^{n} f[m]$	$\left(\frac{z}{z-1}\right)F(z)$
Time Convolution	$f_1[n]*f_2[n]$	$F_1(z) \cdot F_2(z)$
Frequency Convolution	$f_1[n] \cdot f_2[n]$	$\frac{1}{j2\pi}\oint xF_1(v)F_2\left(\frac{z}{v}\right)v^{-1}dv$
Initial Value Theorem	$f[0] = \lim_{z\to\infty} F(z)$	
Final Value Theorem	$\lim_{n\to\infty} f[n] = \lim_{z\to 1}(z-1)F(z)$	

The Z transform of standard discrete−time functions

f[n]	F(z)
$\delta[n]$	1
$\delta[n-m]$	z^{-m}
$a^n u_0[n]$	$\frac{z}{z-a} \quad \lvert z\rvert > a$
$u_0[n]$	$\frac{z}{z-1} \quad \lvert z\rvert > 1$
$(e^{-naT})u_0[n]$	$\frac{z}{z-e^{-aT}} \quad \lvert e^{-aT}z^{-1}\rvert < 1$
$(\cos naT)u_0[n]$	$\frac{z^2 - z\cos aT}{z^2 - 2z\cos aT + 1} \quad \lvert z\rvert > 1$
$(\sin naT)u_0[n]$	$\frac{z\sin aT}{z^2 - 2z\cos aT + 1} \quad \lvert z\rvert > 1$
$(a^n \cos naT)u_0[n]$	$\frac{z^2 - az\cos aT}{z^2 - 2az\cos aT + a^2} \quad \lvert z\rvert > a$
$(a^n \sin naT)u_0[n]$	$\frac{az\sin aT}{z^2 - 2az\cos aT + a^2} \quad \lvert z\rvert > a$
$u_0[n]-u_0[n-m]$	$\frac{z^m - 1}{z^{m-1}(z-1)}$
$nu_0[n]$	$z/(z-1)^2$
$n^2u_0[n]$	$z(z+1)/(z-1)^3$
$[n+1]u_0[n]$	$z^2/(z-1)^2$
$a^n nu_0[n]$	$(az)/(z-a)^2$
$a^n n^2 u_0[n]$	$az(z+a)/(z-a)^3$
$a^n n[n+1]u_0[n]$	$2az^2/(z-a)^3$

$$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos(nt) + \sum_{n=1}^{\infty} b_n \sin(nt)$$

$$C_n = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-jn\omega_0 t}dt$$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega)e^{j\omega t}d\omega$$

$$L[x(t)] = X(s) = \int_{0^-}^{\infty} x(t)e^{-st}\,dt$$

$$x(t) = L^{-1}[X(s)] = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}\,ds$$

$$X(z) = \sum_{n=0}^{\infty} x(nT)z^{-n} \quad where \quad z = e^{sT}$$

$$f[n] = \frac{1}{j2\pi}\oint F(z)z^{k-1}dz$$

$$\int xsinaxdx = \frac{1}{a^2}sinax - \frac{x}{a}cosax$$